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Is the gap anisotropy in high-T_c superconductors really as high as it is commonly believed?

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Abstract

Making use of the photoemission data on the dependence of the order parameter Δ on the in-plane momentum $\boldsymbol{k}$ and of the tight-binding fit to the Fermi surfaces in hole-doped high-T_c superconductors, we have calculated the anisotropy parameter χ which enters into the formula for T_c versus impurity concentration in an anisotropic s wave superconductor. We have found that χ is as low as ~ 0.1 in a wide range of hole doping and for different sets of model parameters adjusted to describe the experimental observations of $\Delta(\boldsymbol{k})$ and Fermi surface in $Bi_2Sr_2CaCu_2O_{8+\delta}$. Our results reconcile the large anisotropy of Δ in $\boldsymbol{k}$ space (i.e. not a simple s wave) with the weak sensitivity of T_c to defects and structural inhomogeneities (i.e. probably not a $d_{x^2-y^2}$ wave) and are compatible with the anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$ in hole-doped high-T_c superconductors. Experiments [H. Ding et al., Phys. Rev. B 50 (1994) 1333] on electron irradiation of the twin-free single crystal of $YBa_2Cu_3O_{7-\delta}$ are consistent with a value of $\chi \approx 0.3$.

1. Introduction

The symmetry of the superconducting gap $\Delta(\boldsymbol{k})$ within the CuO_2 planes is the key issue in understanding the pairing mechanism in high-T_c copper-oxide superconductors. While there is clear evidence for the conventional BCS-like pairing in electron-doped high-T_c cuprates [1], experimental results on hole-doped materials still remain controversial (see, e.g., Refs. [2–6]).

Although neither the existence of nodes in the gap function nor the phase change over a $\pi/2$ rotation (which both are characteristic of $d_{x^2-y^2}$ symmetry) have not been definitely established, experimental findings give evidence for a highly anisotropic gap in the a–b planes of hole-doped high-T_c superconductors. This conclusion is based mainly on ultra-high-resolution angle-resolved photoemission spectroscopy studies. For example, Kelley et al. [7] have found that $\Delta(\boldsymbol{k})$ in $Bi_2Sr_2CaCu_2O_{8+\delta}$ single crystals is as small as 1–2 meV along Γ–Y, 4–8 meV along Γ–X, and 14–20 meV along Γ–M (Cu–O direction in real space). According to Ding et al. [8], $\Delta(\boldsymbol{k})$ in $Bi_2Sr_2CaCu_2O_{8+\delta}$ is minimized, with a value close to zero, along Γ–X and Γ–Y, while $\Delta = 22$ meV along Γ–M. These and many other measurements are consistent with either a $d_{x^2-y^2}$ wave or an anisotropic s wave superconducting state in hole-doped cuprates.

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To distinguish a $d_{x^2-y^2}$ wave from an anisotropic s wave, one can, in principle, concentrate on differences in physical properties of superconductors with different symmetry of $\Delta(\boldsymbol{k})$: magnetic penetration depth, nuclear magnetic resonance, etc. One possible way, which we adopt in this paper, is to quantify the expected sensitivity of the critical temperature T_c to nonmagnetic impurities for d wave and s wave models [9] and to compare the results with the current experimental situation.

We should note that the approach used in Ref. [9] is a Fermi-liquid-based theory. It is a matter of controversy whether this is really legitimate for copper oxides (see, e.g. Ref. [10]), as strong correlation effects play a key role in these compounds and Fermi liquid picture might not be applicable here. Thus, the standard impurity-scattering theory for superconductors may not be the last word on this issue, but rather a temporary basis for discussion.

2. Nonmagnetic impurities in $d_{x^2-y^2}$ wave superconductors

Anderson's theorem [11] tells us that T_c of an isotropic s wave superconductor is not affected by nonmagnetic impurities, at least if the scattering rate (inverse relaxation time) τ^{-1}, which is proportional to the impurity concentration, does not exceed the Fermi energy E_F (throughout this paper we set $\hbar = k_B = 1$). On the other hand, T_c of a $d_{x^2-y^2}$ wave superconductor is strongly suppressed by a minute concentration of nonmagnetic impurities, a characteristic critical value of τ^{-1} being of the order of T_{c0}, the critical temperature of a clean sample.

The form of the T_c (τ^{-1}) curve for a $d_{x^2-y^2}$ wave superconductor is given implicitly by the relation (see, e.g., Ref. [9])

$$\ln\left(\frac{T_{c0}}{T_c}\right) = \psi\left(\frac{1}{2} + \frac{1}{4\pi\tau T_c}\right) - \psi\left(\frac{1}{2}\right), \qquad (1)$$

where $\psi(x)$ is the digamma function. (Hereafter we restrict ourselves to isotropic impurity scattering.) Note that τ^{-1} in Eq. (1) is the *renormalized* scattering rate, $\tau^{-1} = (1+\lambda)^{-1}\tau_0^{-1}$, where λ is the mass-renormalization constant due to the pairing bosons, and τ_0^{-1} is the bare scattering rate [9]. We stress that the experimentally measured resistivity ρ_0 is determined by the value of τ^{-1}, not τ_0^{-1}, thus facilitating contact with experiment [9]. Moreover, as was shown in Ref. [9], the analytic form of the scaling function T_c/T_{c0} versus $(\tau T_{c0})^{-1}$, Eq. (1), is very close to the computer solutions of the Eliashberg equations for both weak ($\lambda \ll 1$) and strong ($\lambda > 1$) coupling. Hence, Eq. (1) with the renormalized scattering rate τ^{-1} can be used to predict the response of a $d_{x^2-y^2}$ wave superconductor to nonmagnetic impurities.

The initial decrease of T_c is described by the formula

$$\frac{T_c}{T_{c0}} = 1 - \frac{\pi}{8}\frac{1}{\tau T_{c0}} \qquad (2)$$

which follows from Eq. (1) in the limit $(4\pi\tau T_{c0})^{-1} \ll 1$. The superconductivity vanishes ($T_c = 0$) at the critical value $(\tau_c T_{c0})^{-1} = 1.76$. Thus, for the $d_{x^2-y^2}$ wave superconductor with $T_c \approx 100$ K we have $\tau_c^{-1} \approx 20$ meV. Radtke et al. [9] represented the scattering rate τ^{-1} in terms of the planar residual resistivity ρ_0 and predicted that the initial slope $dT_c/d\rho_0$ should be $-(0.8 \div 1.2)$ K/($\mu\Omega$ cm), and the critical value ρ_{0c} for the destruction of $d_{x^2-y^2}$ wave superconductivity should be $(50 \div 85)$ $\mu\Omega$ cm. (The uncertainties of $dT_c/d\rho_0$ and ρ_{0c} reflect the uncertainty in the experimentally measured plasma frequency ranging from 1.1 to 1.4 eV in $YBa_2Cu_3O_7$).

Recently, Giapintzakis et al. [12] have examined how T_c and $\rho(T)$ change as a function of the electron-irradiation dose and found that $dT_c/d\rho = -(0.30 \pm 0.04)$ K/($\mu\Omega$ cm), thus ruling out isotropic s wave paring. Though the results of Ref. [12] seem to contradict the prediction of Radtke et al. [9], one should keep in mind that Radtke et al. considered the case of isotropic impurity scattering. As was mentioned in Ref. [12], experimental results are roughly consistent with a d wave superconductor if the impurity scattering has a strong d wave component. At present, however, we are not aware of any systematic experiments on the scattering anisotropy in high-T_c superconductors. So, it is interesting to study in some more detail an alternative way to explain the data of Ref. [12], namely, to consider a suppression of T_c by nonmagnetic impurities in an anisotropic s wave superconductor.

3. Nonmagnetic impurities in anisotropic s wave superconductors

In the case of anisotropic s wave pairing (but isotropic scattering) the functional form of the $T_c(\tau^{-1})$ curve is given by the expression [13,14]

$$\ln\left(\frac{T_{c0}}{T_c}\right) = \chi\left[\psi\left(\frac{1}{2}+\frac{1}{4\pi\tau T_c}\right) - \psi\left(\frac{1}{2}\right)\right], \quad (3)$$

where the coefficient χ is nongeneric. For example, if a circular Fermi surface is considered, $\chi = 1 - 8/\pi^2 \approx 0.19$ for the gap $\Delta(\varphi) = \Delta_0 |\cos(2\varphi)|$ [13], and $\chi = 0.25$ for the gap $\Delta(\varphi) = \Delta_0 |1-(4/\pi)\varphi|$, $0 \leqslant \varphi \leqslant \pi/2$, periodically continued to the interval $[\pi/2, 2\pi]$ [14].

However, a circular Fermi surface is a poor approximation for high-T_c superconductors because of their highly anisotropic electronic structure. In what concerns the dependence of Δ on φ, one can avoid an ambiguity by making use of the photoemission data. A general formula for the parameter χ in the two-dimensional case was given in Ref. [15]:

$$\chi = 1 - \frac{\langle \Delta \rangle^2}{\langle \Delta^2 \rangle}, \quad (4)$$

where

$$\langle \dots \rangle = \frac{\oint (\dots) \frac{dl}{v(\boldsymbol{n})}}{\oint \frac{dl}{v(\boldsymbol{n})}}; \quad (5)$$

$\boldsymbol{n}$ is a unit vector along the momentum, $v(\boldsymbol{n})$ is the absolute value of the planar quasiparticle velocity, the integration is taken along the Fermi contour (i.e. two-dimensional Fermi surface). Thus, the value of χ is determined by the degree of the order-parameter anisotropy. In particular, $\chi = 0$ if $\Delta =$ const. on the Fermi surface (isotropic s wave), and $\chi = 1$ for the $d_{x^2-y^2}$ wave ($\langle \Delta \rangle = 0$). For the anisotropic s wave the parameter χ can take any value between 0 and 1, depending on the actual anisotropy of the superconducting gap.

From a comparison of Eqs. (1) and (3), one can see that T_c of the anisotropic s wave superconductor can be also strongly suppressed by nonmagnetic impurities if the value of the anisotropy parameter χ is large enough. For example, in the case $\chi \sim 1$, the initial sensitivity of the anisotropic s wave to impurities is actually the same as that of the $d_{x^2-y^2}$ wave (the difference between the two consists in the asymptotic approach of T_c to zero for the anisotropic s wave with $\chi \neq 1$ [15], whereas T_c vanishes at some finite $\tau^{-1} \approx T_{c0}$ for the $d_{x^2-y^2}$ wave). For the isotropic s wave ($\chi = 0$) we have $T_c =$ const (Anderson theorem [11]). The initial decrease of T_c at $(4\pi\tau T_{c0})^{-1} \ll 1$ is described by the formula (cf. with Eq. (2))

$$\frac{T_c}{T_{c0}} = 1 - \frac{\pi}{8}\frac{\chi}{\tau T_{c0}}. \quad (6)$$

We emphasize that formulas (3-5) take into account both the anisotropy of the Fermi surface and the in-plane anisotropy of the superconducting gap $\Delta(\boldsymbol{k})$ (or, equivalently, $\Delta(\varphi)$).

At first sight, the strong variation of $|\Delta(\boldsymbol{k})|$ from the Γ-X and Γ-Y to the Γ-M direction, as observed in photoemission experiments [7,8], favors large enough (i.e. close to unity) values of χ, irrespective of the presence or the absence of the sign change of $\Delta(\boldsymbol{k})$ over a $\pi/2$ rotation, i.e. irrespective of $d_{x^2-y^2}$ wave or anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$. Hence, as the condition $T_{c0} \ll E_F$ is obviously fulfilled in high-T_c superconductors, one could expect from Eq. (2) or Eq. (6) a rapid decrease in T_c provided the superconducting state had the $d_{x^2-y^2}$ symmetry or the highly anisotropic s wave symmetry with $\chi \sim 1$.

However, we know that the values of T_c in high-T_c materials do not depend strongly on the sample quality, i.e on the concentration of defects and nonmagnetic impurities. To reconcile this fact with the experimentally observed [7,8] anisotropy of $\Delta(\boldsymbol{k})$, we have calculated the value of χ making use of

(1) photoemission data on the dependence of Δ on $\boldsymbol{k}$ [7], and

(2) the results of the tight-binding fit to the Fermi surfaces in high-T_c superconductors [16].

Some details of our calculations are given below.

We use the form of an order parameter proposed in Ref. [7]:

$$\Delta(\varphi) = \gamma_1 \cos(2\varphi) + i\gamma_2 \cos(\varphi + \varphi_0), \quad (7)$$

where φ is the angle with respect to the Γ–M direction, and the parameters γ_1, γ_2, and φ_0 are chosen to fit the measured $\Delta(\boldsymbol{k})$ values along the major symmetry directions in the Brillouin zone. According in Ref. [7], the majority of the samples were described by the following parameter values:

$$\gamma_1 = 13.1 \text{ meV}, \; \gamma_2 = 6.09 \text{ meV}, \text{ and } \varphi_0 = 118.0°. \tag{8}$$

We would like to stress that the form (7) is nothing more than just the fit to the experimental data on $|\Delta(\varphi)|$, so the presence of terms with $d_{x^2-y^2}$ and $d_{xz}+d_{yz}$ symmetry in Eq. (7) should not be taken too seriously. On an equal footing, the superconducting order parameter in the form (7) may be viewed just as the anisotropic s wave if we deal with the absolute value of Δ. So, assuming the anisotropic s wave order parameter, we set Δ in Eq. (4) to be equal to the absolute value of $\Delta(\varphi)$ given by Eq. (7).

Next, for the description of the Cu–O dpσ antibonding band crossing E_F, we use the simple tight-binding band on a 2D square lattice:

$$\varepsilon(\boldsymbol{k}) = \varepsilon_0 - 2t[\cos(k_x a) + \cos(k_y a)] + 4t' \cos(k_x a) \cos(k_y a), \tag{9}$$

where the parameters ε_0, t, and t' are chosen to fit the Fermi surfaces in hole-doped cuprates [16] (as opposed to parent insulators). The knowledge of the absolute values of the parameters, ε_0, t, and t' is not necessary for our purposes. We need only the relative value t'/t which is equal to 0.45 [16].

Now we proceed with the calculation of the anisotropy parameter χ. First, fixing the number p of doped holes per copper site ($p = 0$ at half filling, i.e. $p = 1 - n$, where n is the number of electrons per site), we determine the Fermi contour on a 2D lattice. Then we average the values $|\Delta|$ and $|\Delta|^2$ over this contour, $|\Delta|$ being taken in the form (7) with the parameter set (8), and the Fermi velocity $v(\boldsymbol{n})$ in Eq. (5) being simply $[(\partial\varepsilon(\boldsymbol{k})/\partial k_x)^2 + (\partial\varepsilon(\boldsymbol{k})/\partial k_y)^2]^{1/2}$. As a result, from Eq. (4) we obtain the value of χ at a given doping level p.

We have calculated the anisotropy parameter χ in the range $p = 0.1$–0.3, roughly corresponding to the superconducting part of the T_c–p phase diagram of high-temperature superconductors. The most surprising result is that the actual values of χ appeared to be almost an order of magnitude smaller than we have expected from the apparently high anisotropy of Δ in $\boldsymbol{k}+$ space (7). Namely, $\chi = 0.122$–0.126 in the full range of p values studied. It is interesting that the dependence of χ on p is very weak.

To check whether the small χ values arise just from the specific values of the parameters γ_1, γ_2, φ_0 in Eq. (7) and t', t in Eq. (9), we have calculated χ for two other parameter sets in Eq. (7):

$$\gamma_1 = 19.8 \text{ meV}, \; \gamma_2 = 2.82 \text{ meV}, \text{ and } \varphi_0 = 0.0°, \tag{10}$$

and

$$\gamma_1 = 12.6 \text{ meV}, \; \gamma_2 = 16.1 \text{ meV}, \text{ and } \varphi_0 = 15.3°, \tag{11}$$

as well as for different values of the t'/t ratio in Eq. (9). The set (10) obviously corresponds to a larger anisotropy of $\Delta(\boldsymbol{k})$ as compared with the set (8), since the ratio γ_1/γ_2 for Eq. (10) is much larger than that for Eq. (8). The set (10) is taken from Ref. [7] where it has been proposed as a fit to the photoemission data of Shen et al. [17]. On the other hand, the set (11) clearly points to a smaller degree of gap anisotropy. This set has been found in Ref. [7] for a small fraction of samples studied (though the origin of the parameters variation has not been completely understood in Ref. [7], we would like to stress that a similar effect, i.e. the more isotropic superconducting gap, has been also observed in some samples by Ding et al. [8]).

Our results for the case $t'/t = 0.45$ are as follows: $\chi = 0.178$–0.185 for the set (10), and $\chi = 0.071$–0.072 for the set (11), in accordance with the apparent difference in the gap anisotropy in $\boldsymbol{k}$ space given evidence for by the relative values of γ_1 and γ_2 in Eqs. (8), (10), and (11). In all three cases, the value of χ decreases slightly under hole doping. It is worthwhile to note that the dependence of χ on p weakens with the lowering of the characteristic value of χ in the full range of hole doping studied. We have also calculated the value of χ for various t'/t ratios, and again did not find any appreciable variation.

So, the value of the anisotropy parameter χ is roughly an order of magnitude smaller ($\chi \sim 0.1$) than one could expect from the strong variation of Δ

in $\boldsymbol{k}$ space. But it is the parameter χ that enters into the formulas (3) and (6) for the dependence of T_c on the concentration of nonmagnetic impurities in anisotropic s wave superconductors. Hence, the small values of χ in hole-doped high-T_c superconductors give evidence for the anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$, as they reconcile the strong anisotropy of Δ in $\boldsymbol{k}$ space (i.e. definitely not a simple s wave) with the weak sensitivity of T_c to defects and structural inhomogeneities (i.e. probably neither a $d_{x^2-y^2}$ wave nor an anisotropic s wave with $\chi \sim 1$).

4. Comparison with experiment

It follows from Eqs. (2) and (6) that the initial rate of T_c decrease appears to be an order of magnitude smaller in the anisotropic s wave superconductor with $\chi \sim 0.1$ than in the $d_{x^2-y^2}$ wave superconductor with the same value of T_{c0}. As was shown by Radtke et al. [9], strong-coupling effects have no appreciable influence on the dependence of T_c on τ^{-1} if one uses the renormalized value of τ^{-1} which is proportional to the experimentally measured planar resistivity ρ_0. Following the line of arguments given in Ref. [9], we have converted the $T_c(\tau^{-1})$ dependences into $T_c(\rho_0)$ dependences for different values of the anisotropy parameter χ. The results are presented in Fig. 1 for a 100 K superconductor with the plasma frequency $\omega_{pl} = 1$ eV. This choice of T_{c0} and ω_{pl} is, to some extent, arbitrary. In order to go to the other values of T_{c0} and ω_{pl} one should replace ρ_0 in Fig. 1 by $\rho_0(T_{c0}/100)\omega_{pl}^{-2}$, where T_{c0} is measured in K, and ω_{pl} is measured in eV (see Ref. [9] for details).

It follows from Fig. 1 that $T_{c0} - T_c \sim 1$ K if $\chi \sim 0.1$ and $\rho_0 \sim 20$ μΩ cm. This is in apparent agreement with experimental observations that T_c of high-temperature superconductors do not depend strongly on sample quality, being nearly the same for ceramics, single crystals, and thin films. By contrast, if copper oxides were $d_{x^2-y^2}$ wave superconductors, then T_c would be lowered by ~ 10 K at $\rho_0 \sim 20$ μΩ cm (see Fig. 1), in striking conflict with the experiment. Moreover, the variation of T_c by ~ 1 K from sample to sample is typical for all high-T_c cuprates (including high-quality crystals) and reflects the difficulties in controlling the minute defect concentrations and structural imperfections (on the other hand, T_c's of conventional superconductors with isotropic s wave symmetry of $\Delta(\boldsymbol{k})$ are known with accuracy ~ 0.1 K).

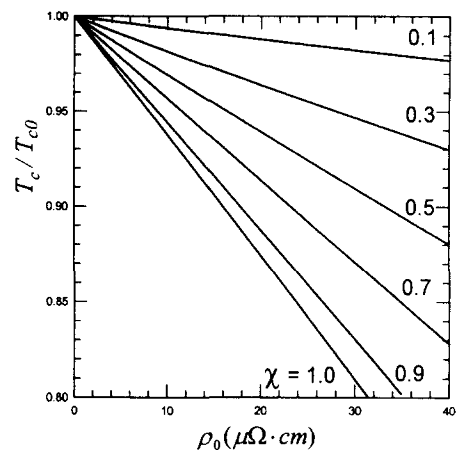

Fig. 1. Normalized critical temperature T_c / T_{c0} as a function of the in-plane residual resistivity ρ_0 due to nonmagnetic impurities in a 100 K superconductor with a d wave ($\chi = 1$) or an anisotropic s wave ($\chi = 0.1, 0.3, 0.5, 0.7, 0.9$) order parameter. The plasma frequency is chosen to be $\omega_{pl} = 1$ eV. One can easily go to the other values of T_{c0} and ω_{pl} through replacing ρ_0 by $\rho_0(T_{c0}/100)\omega_{pl}^{-2}$, where T_{c0} is measured in K, and ω_{pl} is measured in eV [9].

Fig. 2 shows the dependences of T_c/T_{c0} on ρ_0 for 90 K superconductors with $\chi = 1$ and 0.3 along with the experimental dependence [12] of T_c/T_{c0} of $YBa_2Cu_3O_{7-\delta}$ untwinned single srystal on the difference between the values of $\rho(T = 145$ K) in electron-irradiated and -unirradiated samples. The theoretical curves were computed for plasma frequencies $\omega_{pl} = 1.1$ and 1.4 eV. This choice of ω_{pl} reflects the range of experimental uncertainty in ω_{pl} in $YBa_2Cu_3O_{7-\delta}$ (see Refs. [9] and [12]).

From Fig. 2 one can see that experimental data are consistent with the anisotropic s wave superconductivity with the gap anisotropy parameter $\chi = 0.3$. This value of χ is a factor of 2 to 3 larger than that calculated above for the compound $Bi_2Sr_2CaCu_2O_{8+\delta}$. However, it is not clear to what extent the change of $\rho(T = 145$ K) under irradiation reflects the change of ρ_0. As it was mentioned in Ref. [12], the extrapolation of (T) to $T = 0$ gives negative values of ρ_0, so the resistivity in the absence of superconductivity could not remain linear down to

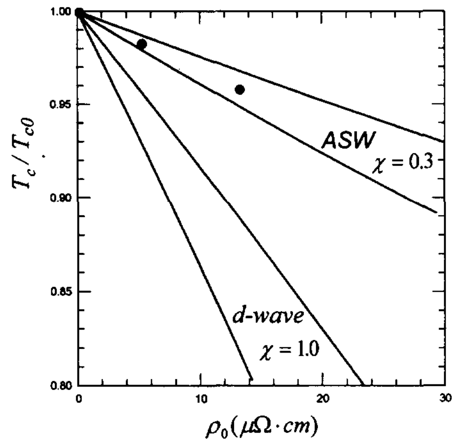

Fig. 2. T_c/T_{c0} vs. ρ_0 in a 90 K superconductor with a d wave or an anisotropic s wave (ASW) order parameter ($\chi = 0.3$) and plasma frequencies ω_{pl} ranging from 1.1 to 1.4 eV (upper and lower curve for a given value of χ). This choice of ω_{pl} reflects the range of experimental uncertainty in ω_{pl} in $YBa_2Cu_3O_7$. The solid points show the experimental data on an electron-irradiated untwinned single crystal of $YBa_2Cu_3O_7$ taken from Ref. [12].

$T = 0$. Thus, the change in $\rho(T = 145$ K) under irradiation may be appreciably different from the corresponding change in ρ_0. Moreover, the choice of $\rho_0 = 0$ in the unirradiated sample adopted in Ref. [12] and used by us in Fig. 2 is probably not correct, as even "high-quality" crystals have more or less impurities and structural imperfections. Taking this fact into account will result in shifting the experimental points of Fig. 2 to the right side, thus leading to a lower value of χ, closer to that in $Bi_2Sr_2CaCu_2O_{8+\delta}$. It would be interesting to conduct experiments on irradiation of high-temperature superconductors with low T_{c0} values, e.g., $Bi_2Sr_2CuO_{6+\delta}$ with properly adjusted oxygen content, in order to make possible a direct determination of the residual resistivity ρ_0.

Besides, the data of Ref. [12] are confined to very low irradiation doses, so that the change in T_c under irradiation is comparable to or less than the width of the resistive transition ΔT_c. This gives rise to an additional error stemming from the uncertainty in $T_{c0} - T_c$. Thus, it would be interesting to broaden the range of $T_{c0} - T_c$ values, keeping the value of ΔT_c roughly unchanged.

In what concerns speculations [12] about a possible strong influence of the scattering anisotropy on the T_c versus ρ_0 curve, we want to note that for a dimensionless constant g_I which enters into the equations for T_c [18], in general, one expects $|g_I| \ll 1$, i.e. very close to the value of $g_I = 0$ for isotropic impurity scattering. On the other hand, it was shown in Ref. [12] that experimental data would be roughly consistent with a d wave superconductor with $g_I = 0.5$. In our opinion, such a large value of g_I (i.e. such a strong d wave component in the impurity scattering) in high-temperature superconductors is unlikely.

5. Discussion and conclusions

Finally, we would like to highlight the physical reason for the small values of the anisotropy parameter χ in hole-doped high-T_c superconductors, which seems to be in a contradiction with the strong anisotropy of Δ in $\boldsymbol{k}$ space. We emphasize that the actual degree of the gap anisotropy is, in fact, determined not only by the dependence of Δ on $\boldsymbol{k}$, but also by the specific form of the Fermi surface. For example, as pointed out by Li et al. [19], in the case of the single-particle dispersion $\varepsilon(\boldsymbol{k}) = -2t[\cos(k_x a) + \cos(k_y a)]$, the extended s wave $\Delta(\boldsymbol{k}) = \Delta_0[\cos(k_x a) + \cos(k_y a)]$ is essentially equivalent to the usual isotropic s wave state $\Delta(\boldsymbol{k}) = \Delta_0 =$ const since the extended s wave state is also constant on the Fermi surface (and we obviously have $\chi = 0$). So, low values of χ in our calculations arose from the weak variation of Δ along the two-dimensional Fermi surface described by Eq. (9), despite the fact that the dependence of Δ on $\boldsymbol{k}$ was rather strong (7). Moreover, the weak influence of the hole-doping level p on the *form* (as opposed to the *area*) of the Fermi surface resulted in a weak dependence of χ on p.

Also, it is interesting to calculate the values of the anisotropy parameter χ for some model dependences $\Delta(\boldsymbol{k})$ or $\Delta(\varphi)$ used in theoretical studies of anisotropic s wave superconductivity. We have computed the values of χ for the cases

(1) $\Delta(\varphi) = \Delta_0|1 - (4/\pi)\varphi|$. $0 \le \varphi \le \pi/2$, periodically continued to the interval $\pi/2 \le \varphi \le 2\pi$ [14];

(2) $\Delta(\varphi) = \Delta_0|\cos(2\varphi)|$ [13];

(3) $\Delta(\boldsymbol{k}) = \Delta_0[\cos(k_x a) + \cos(k_y a)]$ [20], making use, as before, of the tight-binding model (5), with $t'/t = 0.45$ and taking the concentration p of doped

holes to be equal to 0.2 (this choice of p roughly corresponds to the maximum of T_c in the T_c–p phase diagram). Our results are as follows:
(1) $\chi = 0.26$.
(2) $\chi = 0.20$.
(3) $\chi = 0.47$.

It is worth noting that in all three cases considered the anisotropy parameter χ varies slightly with p over the interval $0.1 \leq p \leq 0.3$, as in the case of phenomenological $\Delta(\varphi)$ dependences considered above.

We now discuss the case of extended s wave order parameter $\Delta_{es}(\boldsymbol{k}) = \Delta_0[\cos(k_x a) + \cos(k_y a)]$ more closely. Recently [20] we have studied the influence of site-diagonal Anderson disorder (analogue of non-magnetic impurities) on pairing correlators p_d and p_{es} with d wave and extended s wave symmetries through an exact diagonalization of the Emery hamiltonian on a Cu_4O_8 cluster. We have shown that both p_d and p_{es} decrease with the strength of disorder W (which is proportional to defect concentration), but the effect of disordering on p_d is much more severe than on p_{es}. Pairing correlations in the d wave channel were shown to vanish at a critical value $W_c^d \approx 2t$. However, in Ref. [20] we were unable to determine the corresponding value W_c^{es} in extended s wave channel, though we have noted that W_c^{es} is greater than W_c^d (see Ref. [20] for more details). Now, based on the results obtained in this paper, we may conclude that the initial (i.e. at low defect concentration) sensitivity of the d wave to disordering is twice as high as that of the extended s wave, since $\chi = 1$ and ≈ 0.5 for a d wave and an extended s wave, respectively. Besides, upon an increase of defect concentration the superconducting state with $\chi < 1$ becomes less sensitive to the disorder as compared to the d wave state with $\chi = 1$ [15]. Hence,one would expect that W_c^{es} in cluster calculations exceeds W_c^d more than by a factor of two, i.e. $w_c^{es} > 4t$, in a qualitative correspondence with the results of Ref. [20].

In summary, based on the photoemission data for the dependence of the order parameter Δ on $\boldsymbol{k}$ and on the tight-binding fit to the Fermi surfaces in hole-doped high-T_c superconductors, we have calculated the anisotropy parameter χ which enters into the formula for T_c versus impurity concentration in an anisotropic s wave superconductor. We have found that χ is as low as ~ 0.1 in the wide range of model parameters and hole doping $p = 0.1$–0.3. Our results reconcile the large anisotropy of Δ in $\boldsymbol{k}$ space (i.e. not a simple s wave) with the weak sensitivity of T_c to defects and structural inhomogeneities (i.e. not a $d_{x^2-y^2}$ wave) and are compatible with the anisotropic s wave symmetry of $\Delta(\boldsymbol{k})$ in hole-doped high-T_c superconductors. The experimental data on electron irradiation of an untwinned single crystal of $YBa_2Cu_3O_{7-\delta}$ are consistent with a value of $x \approx 0.3$, though the actual wave of χ may be 0.1–0.2 because of uncertainties in $T_{c0} - T_c$, residual resistivity and plasma frequency.

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